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We find a formulation of hard diffraction unifying the partonic (Ingelman-Schlein) Pomeron, Soft Colour Interaction and QCD dipole descriptions. A theoretical interpretation in terms of S-Matrix and perturbative QCD properties in the small x_{Bj} regime is proposed.

1. In the present paper, we focus on three existing different theoretical approaches of hard diffraction, for which we propose a new, unifying, formulation. The first one we will refer to is the "partonic Pomeron" approach [1]. The hard photon is here supposed to probe the parton structure of the Pomeron considered as an hadronic particle. A second approach [2] is the Soft Colour Interaction model, where hard diffraction is described as the superposition of two processes. At short time, the hard probe initiates a typical deep-inelastic interaction with colour quantum numbers exchange. Then, at large times/distances, a "soft" colour interaction is assumed to rearrange the colour quantum numbers and give rise to singlet exhanges -and thus diffraction- with a probability of order $\frac{\mathbf{1}}{\mathbf{N_c^2}}$, where N_c is the number of colours[1] . A third approach is based on an extension [3] to hard diffractive processes of the "QCD dipole" approach in the small x_{Bj} regime of perturbative QCD [4].

In the present paper, we show that these three aproaches may find a common formulation. Our main results are the following:

i) The "effective" parameters of the partonic Pomeron are determined from leading log perturbative QCD resummation. They are found to depend not only on Q^2 but also on the ratio $\eta = (Y-y)/y$, where Y (resp. y) are the total (resp. gap) rapidity interval.

We obtain

$$F_{T,L}^{Diff}(Q^2,Y,y) \approx \frac{\mathbf{1}}{\mathbf{N_c^2}}\,\frac{\mathcal{N}^{tot}}{x_P}\,\frac{e^{2y\Delta}}{4\pi\Delta'' y}\sqrt{\frac{2}{1+2\eta}}\exp\left\{(Y-y)\,\epsilon_s\right\}\left(\frac{Q}{Q_0}\right)^{2\gamma_s}\exp\left(-\frac{2\log^2\left(\frac{Q}{Q_0}\right)}{D_s(Y-y)}\right), \qquad (1)$$

with

$$\Delta(x) \equiv \frac{\alpha_s N_c}{\pi}\left\{2\psi(1)-\psi(x)-\psi(1-x)\right\} \sim \Delta + \frac{\Delta''}{2}\,(1/2-x)^2 \qquad (2)$$

is the BFKL (Balitsky, Fadin, Kuraev, Lipatov) evolution kernel [4] (together with its gaussian approximation near the minimum at $x = 1/2$) and Q_0 a non-perturbative scale associated with the proton. We have

$$\gamma_{\mathbf{s}} = \frac{\eta}{1+2\eta}\ ;\ \epsilon_s = \Delta + \frac{\Delta''}{8(1+2\eta)}\ ;\ D_s = \frac{1+2\eta}{\eta}\,\Delta''\ . \qquad (3)$$

One may write $F_{T,L}^{Diff} \sim \sigma_{\gamma^*-P}^{tot} \times e^{2y\Delta}/(4\pi\Delta'' y)$, which is the known "triple Pomeron" formula [9] where $\sigma_{\gamma^*-!P}^{tot}$ defines the effective interaction cross-section of a virtual photon with a BFKL Pomeron $e^{y\Delta}/\sqrt{4\pi\Delta'' y}$ derived from the QCD dipole formalism. By analogy with BFKL [4], γ_s, ϵ_s and D_s can be defined, respectively, as the anomalous dimension, intercept and diffusion parameter of an "effective" BFKL $\gamma^* - P$ cross-section. Quite interestingly, the unknown normalization in the QCD dipole model description is now determined as the product of the factor $\frac{\mathbf{1}}{\mathbf{N_c^2}}$ and the normalization factor $\mathcal{N}^{tot}$ of the non-diffractive structure function, according to the Soft Colour Interaction principle.

ii) In Soft Color Interactions models, one expects the following relation between the total structure function and the overall contribution of hard diffraction at fixed value of x_{Bj} :

$$F_{T,L}^{Diff/tot} \equiv \int_{x_{Bj}}^{x_{gap}} dx_P\ F_{T,L}^{Diff} = \frac{\mathbf{1}}{\mathbf{N_c^2}}\ F_{T,L}^{tot} \qquad (4)$$

where $\log 1/x_{gap}$ is the minimal rapidity gap. If we insert the QCD dipole prediction for $F_{T,L}^{Diff}$ in formula (4), we find

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[1]More recent versions of the model consider a probability factor of the same order but not necessarily connected with colour.

$$F_{L,T}^{tot} \approx \frac{\mathcal{N}}{\mathbf{N_c^2}} \left(\frac{Q}{Q_0}\right)^{2\gamma^*} \frac{\exp\left(Y\Delta(\gamma^*)\right)}{\sqrt{2\pi\Delta'' \ Y}} \ , \tag{5}$$

which is equivalent to a canonical BFKL expression for non-diffractive structure functions, apart the substitution of the BFKL effective anomalous dimension

$$\gamma_{BFKL} = \ 1/2 - 2\frac{\log\left(\frac{Q}{Q_0}\right)}{\Delta'' \ Y} \ \ \rightarrow \ \gamma^* = cst. \sim 0.175 \ , \tag{6}$$

where the "universal" value γ^* is solution (for $0 < \gamma < 1/2$) of the implicit equation $2\Delta\left(\frac{1-\gamma}{2}\right) - \Delta(\gamma) = 0$. Here, it is interesting to note that the shift (6) may be useful to avoid the objection [5] based on the "Low's theorem" following which soft colour radiation cannot be emmitted from inside a partonic process. The differences we find with the original model means that, in our dipole formulation, the soft colour interaction indeed seems to modify the initial parton kinematics.

iii) Using S-Matrix properties of triple-Regge contributions, a relation is found between discontinuities of a $3 \rightarrow 3$ amplitude and the three approaches to hard diffraction we consider. Following old results of S-Matrix theory in the Regge domain [10], and as sketched in Fig.1, one may consider three types of discontinuities of a $3 \rightarrow 3$ amplitude representing hard diffraction. A single discontinuity over the diffractive $mass^2$ variable for the partonic Pomeron model, a double discontinuity taking into account the analytic discontinuity in the subenergy variable of one of the incident Pomeron exchanges for the Soft Colour Interaction model and the full triple discontinuity with the discontinuity including the two Pomeron exchanges, which is characteristic of the QCD dipole model description [3]. An interesting new feature is the S-Matrix interpretation of the Soft Colour Interaction approach as a specific double discontinuity the $3 \rightarrow 3$ forward amplitude, which formulates the model in terms of simultaneous exchanges of a soft and a hard Pomeron.

2. Let us sketch the derivation of our results.

Our starting point is a triple-Regge formula for the (3-dimensional) structure function for longitudinal and transverse photon in the QCD dipole formalism:

$$F_{T,L}^{Diff}(Q^2, Y, y) \sim \frac{\mathcal{N}^{Diff}}{x_P} \int_{c-i\infty}^{c+i\infty} \frac{d\gamma_1}{2i\pi}\frac{d\gamma_2}{2i\pi}\frac{d\gamma}{2i\pi} \ \delta(1-\gamma_1-\gamma_2-\gamma) \ \left(\frac{Q}{Q_0}\right)^{2\gamma} \ \exp\left\{y(\Delta(\gamma_1)+\Delta(\gamma_2)) + (Y-y) \ \Delta(\gamma)\right\} \ , \tag{7}$$

where $\Delta(\gamma)$ is the BFKL evolution kernel (2) and $\mathcal{N}^{Diff}$ is a normalization containing both QCD perturbative and non-perturbative factors [7]. Strictly speaking [7] the δ-function is the unique contribution in the (4-dimensional) diffractive structure function at momentum transfer $t = 0$. However, it can be shown that this is the dominant perturbative contribution even at non zero transfer due to the pattern of zeroes in the QCD triple-Pomeron couplings [8].

The first step of the computation of formula (7) is to use the saddle-point approximation [7] at large y to integrate over the difference $\gamma_1 - \gamma_2$. One easily gets

$$F_{T,L}^{Diff} = \mathcal{N}^{Diff}\frac{1}{x_P} \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2i\pi} \sqrt{\frac{1}{4\pi\Delta''(\frac{1-\gamma}{2}) \ y}} \ \exp\left\{2y \ \Delta\left(\frac{1-\gamma}{2}\right) + (Y-y) \ \Delta(\gamma) + 2\gamma\log\frac{Q}{Q_0}\right\} \ . \tag{8}$$

Using the gaussian approximation (2) for the BFKL kernels $\Delta(\gamma)$ and $\Delta\left(\frac{1-\gamma}{2}\right)$ in the relevant interval $0 < \gamma < 1/2$, and again a saddle-point approximation at large rapidity gap y, one obtains, up to a normalization factor, formula (1), with γ_s, ϵ_s and D_s defined as in (3). The normalization $\mathcal{N}^{Diff}$ is not yet specified at this stage.

The derivation of the normalization is coming from the comparison with the Soft Colour Interaction approach. Inserting (7) in the integral (4), one is led to perform a two-dimensional saddle-point approximation in the y, γ complex plane. The saddle-point equations read:

$$\begin{aligned} -y \ \Delta^{'}\left(\frac{1-\gamma}{2}\right) + (Y-y) \ \Delta^{'}(\gamma) + 2\log\frac{Q}{Q_0} &= 0 \\ 2\Delta\left(\frac{1-\gamma}{2}\right) - \Delta(\gamma) &= 0 \ , \end{aligned} \tag{9}$$

whose solution (y^*, γ^*) is

$$y^* = \left(Y + \frac{2\log\frac{Q}{Q_0}}{\Delta'(\gamma^*)}\right)\left(1 + \frac{\Delta'\left(\frac{1-\gamma^*}{2}\right)}{\Delta'(\gamma^*)}\right)^{-1}$$

$$\Delta(\gamma^*) = 2\Delta\left(\frac{1-\gamma^*}{2}\right) \tag{10}$$

resulting in a value of $\gamma^* \simeq 0.175$, see which is "universal", i.e. independent of the kinematics of the reaction.

After a tedious but straightforward computation of the prefactors to the saddle-point approximation, one finds:

$$F_{T,L}^{Diff/tot} = \mathcal{N}^{Diff}\frac{1}{|\Delta'(\frac{1-\gamma^*}{2})+\Delta'(\gamma^*)|}\left(\frac{Q}{Q_0}\right)^{2\gamma^*}\frac{\exp\left(Y\Delta(\gamma^*)\right)}{\sqrt{4\pi\Delta''\left(\frac{1-\gamma^*}{2}\right)y^*}}\ . \tag{11}$$

Using (11) and by comparison with the canonical BFKL formula we identify the "hard" interaction as obtained from the substitution $\gamma_{BFKL} \to \gamma^*$, see (6). Then using the Soft colour Interaction ansatz (4), we obtain the relation

$$\mathcal{N}^{Diff} \approx \frac{\mathcal{N}^{tot}}{\mathbf{N_c^2}} \times\ |\Delta'(\frac{1-\gamma^*}{2})+\Delta'(\gamma^*)| \tag{12}$$

which fixes the relative normalization of the diffractive vs. non diffractive structure functions. This ends the derivation of formulae (1- 6).

3. Let us finally come to the S-Matrix interpretation of our approach. For every fixed but arbitrary value of the parameters $\gamma, \gamma_1, \gamma_2$, the triple-Regge formula (7) can be obtained from the canonical formulae corresponding to the vertex of three Regge pole singularities[2] in the complex plane of angular momentum [9]. As such, one can make use of the important S-Matrix Mueller-Regge relation [10], valid in kinematical regions including the triple-Regge limit, between semi-inclusive amplitudes and specific discontinuity contributions of forward elastic $3 \to 3$ amplitudes. It naturally applies to hard diffraction initiated by a virtual photon, as sketched in Fig.1, namely

$$\gamma^* + p \to p + X \quad \Longleftrightarrow \quad Disc_1\left\{\gamma^*\bar{p}\ p\ \to \gamma^*\bar{p}\ p\right\}\ . \tag{13}$$

Quite interestingly, the existence of Regge phase factors allows one to relate other discontinuities of $A(3 \to 3)$ to $Disc_1 A$. As sketched in Fig.1, one may also consider a double discontinuity $Disc_2 A(3 \to 3)$ taking into account also the analytic discontinuity in the subenergy of one of the incident Pomeron exchanges and a triple discontinuity $Disc_3 A(3 \to 3)$ including the discontinuity over the two Pomeron exchanges. The expression of the discontinuities, through generalized unitarity relations, is obtained through the imaginary part of the relevant Regge phase factors [10]. Moreover, one finds an equality relation $Disc_1 A = Disc_2 A = Disc_3 A$ which is due to the fact that the discontinuity taken over the mass variable (corresponding to diffractively produced states) is common to all three cases in Fig.1 and factorizes the same $p\bar{p}$ vertex in $A(3 \to 3)$ (cf. the classical derivation in the last paper of Ref. [10]).

Let us now take advantage of the hard probe in the process, allowing one to introduce in the game the (resummed) perturbative QCD expansion at high energy (small x_{Bj}). In a generic S-Matrix approach, the analytic discontinuities of scattering amplitudes are related to a summation over a complete set of asymptotic *hadronic* final states. If however, the underlying microscopic field theory is at work with small renormalized coupling constant due to the hard probe, it is possible in some cases to approximate the same discontinuity using a complete set of *partonic* states. In particular, at high energy and within the approximation of leading logs (and also large N_c), QCD dipoles can be identified as providing such a basis [3].

While $Disc_1$ and $Disc_3$ correspond to known interpretations of the "partonic Pomeron" and QCD dipole approaches, $Disc_2$ is quite interesting since it appears as a good newe candidate for a description of the Soft Colour Interaction approach, see Fig.1. It appears as a "hard" partonic interaction very similar to the one describing ordinary deep-inelastic processes, in parallel with a "soft" correction evolving during a long time, corresponding to the uncut Pomeron singularity in the middle graph of Fig.1.

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[2]We neglect at this stage the complications due to Regge cuts or other more sophisticated singularities and deal with simple effective Regge poles.

FIGURE

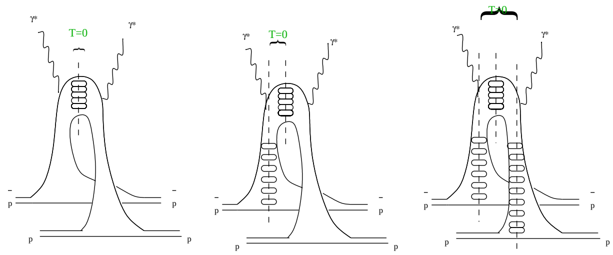

Figure 1
S-Matrix interpretation of the three approaches to hard Diffraction. Upper graph: Description of $Disc_1 A(3 \rightarrow 3)$ (partonic Pomeron approach); Middle graph: Description of $Disc_2 A(3 \rightarrow 3)$ (candidate for the Soft Colour Interaction approach); Lower graph: Description of $Disc_3 A(3 \rightarrow 3)$ (QCD dipole approach).

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